LOYOLA COLLEGE (AUTONOMOUS) CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIFTH SEMESTER – **APRIL 2025**

UMT 5601 – GRAPH THEORY

Date: 08-05-2025 Dept. No. Max. : 100 Marks

Time: 01:00 PM - 04:00 PM

	SECTION A - K1 (CO1)
	Answer ALL the Questions - **(10 x 1 = 10)**
1.	**Answer the following**
a)	When do you say that a graph is simple?
b)	Give an example of Eulerian graph that is not Hamiltonian.
c)	Define eccentricity of a vertex.
d)	Define a cut-set in a connected graph.
e)	Draw two examples of non-planar graphs.
2.	**Fill in the blanks**
a)	The degree of a pendant vertex is ___________.
b)	The number of edges in a complete graph on 5 vertices is ___________.
c)	A connected graph without circuits is known as ___________.
d)	The number of edges in the smallest cut-set is called ___________.
e)	The chromatic number of a tree with n vertices is ___________.
	SECTION A - K2 (CO1)
	Answer ALL the Questions **(10 x 1 = 10)**
3.	**MCQ**
a)	The father of graph theory (i) Gauss (ii) Euler (iii) Fermat (iv) Hamiltonian
b)	For any graph G, the ring-sum $G \oplus G$ (i) G (ii) null graph (iii) $G \cup G$ (iv) $G \cap G$
c)	The number of edges in a tree with n vertices is (i) $n-1$ (ii) n (iii) $n+1$ (iv) $2n-1$
d)	The vertex connectivity of a separable graph is (i) 0 (ii) 1 (iii) n (iv) ∞
e)	In a digraph, the sum of the in-degrees of all vertices is (i) Equal to the number of edges (ii) Double the number of edges (iii) Equal to the sum of the out-degrees (iv) Equal to the number of vertices
4.	**True or False**
a)	The number of vertices of odd degree in a graph is always odd.
b)	The fusion of two vertices reduces the number of edges by one.
c)	In a graph G, there is one and only one path between every pair of vertices is called a tree.
d)	Every cut-set in a connected graph G must contain at least one branch of every spanning tree of G.
e)	The chromatic number of any circuit is three.
	SECTION B - K3 (CO2)
	Answer any TWO of the following **(2 x 10 = 20)**
5.	Show that a simple graph with n vertices and k components can have at most $(n-k)(n-k+1)/2$ edges.

6.	Apply induction method to prove that a tree with n vertices has $n-1$ edges.
7.	Show that a graph of n vertices is a complete graph if and only if its chromatic polynomial is given by $P_n(\lambda) = \lambda(\lambda-1)(\lambda-2)\ldots(\lambda-n+1)$.
8.	Prove that a complete graph on five vertices is nonplanar.
SECTION C – K4 (CO3)	
Answer any TWO of the following	**(2 x 10 = 20)**
9.	Explain that a graph G is disconnected if and only if its vertex set V can be partitioned into two nonempty disjoint subsets V_1 and V_2 such that there exists no edge in G whose one end vertex is in V_1 and the other end in V_2.
10.	Explain the basic operations on graph with suitable examples.
11.	Analyze whether the distance between vertices of a connected graph forms a metric.
12.	Explain that every circuit has an even number of edges in common with any cut-set.
SECTION D – K5 (CO4)	
Answer any ONE of the following	**(1 x 20 = 20)**
13.	(a) Determine that if n is an odd number and $n \geq 3$, there are $(n-1)/2$ edge-disjoint Hamiltonian circuits in a complete graph on n vertices. (b) Check whether a graph G with n vertices, $n-1$ edges, and no circuits is connected. (10+10)
14.	(a) Prove that the ring sum of any two cut-sets in a graph is either a third cut-set or an edge disjoint union of cut-sets. (b) Show that a connected planar graph with n vertices and e edges has $e-n+2$ regions. (10+10)
SECTION E – K6 (CO5)	
Answer any ONE of the following	**(1 x 20 = 20)**
15.	(a) Discuss that a given connected graph G is Euler graph if and only if all vertices of G are of even degree. (b) Demonstrate that every tree has either one or two centers. (10+10)
16.	(a) With respect to a given spanning tree T, prove that a chord c_i determines a fundamental circuit Γ which occurs in every fundamental cut-set associated with the branches in Γ and in no others. (b) Construct a graph by taking any 20 places in our college as vertices and defining connectivity between them as edges. Find the degree of each vertex, and check whether the constructed graph is Eulerian and Hamiltonian. (10+10)

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